GRB 090510: Modeling of Multiwavelength Data

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Detection of GRB 090510

- **Fermi GBM and LAT observations**
  - Trigger on 2009 May 10 at 00:22:59 UT
  - Fluence of the burst: \((T_0+0.5 - T_0+1.0)\) s
    - \(5 \times 10^{-5}\) erg cm\(^{-2}\) (10 keV - 30 GeV); \(4 \times 10^{-7}\) erg cm\(^{-2}\) (15 keV - 150 GeV)

- **Swift BAT observations**
  - Trigger on 2009 May 10 at 00:23:00 UT
  - \((RA, DEC) = (333.55^\circ, -26.58^\circ)\)
  - Duration: \(T_{90} = 0.3 \pm 1\) s
  - Fluence of the burst: \((T_0+0 - T_0+0.4)\) s
    - \(4 \times 10^{-7}\) erg cm\(^{-2}\) (15 keV - 150 GeV)

- **Spectroscopic redshift (3.5 days) from VLT/FORS2**
  - 0.903 ± 0.003
  - \(10^{53}\) erg isotropic-equivalent gamma-ray energy release!
  - Most luminous short GRB detected to-date!!
GBM and LAT Light Curves

Photon arrival info.

- GBM triggered on a weak precursor
- Main GBM emission starts at \( \sim T_0 + 0.5 \) sec
- >100 MeV emission starts at \( \sim T_0 + 0.65 \) s
- >1 GeV emission starts at \( \sim T_0 + 0.7 \) s
- 31 GeV photon at \( \sim T_0 + 0.83 \) s
- Highest from a SGRB
- Extended HE emission

Ackermann et al. 2010
Duration of GRB 090510

Duration varies across the instruments in different energy ranges

<table>
<thead>
<tr>
<th>Instrument</th>
<th>$T_{90}$ (s)</th>
<th>$T_{50}$ (s)</th>
<th>Energy Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBM/NaI 3</td>
<td>0.6</td>
<td>0.2</td>
<td>50–300 keV</td>
</tr>
<tr>
<td>GBM/NaI 6</td>
<td>9.0</td>
<td>0.3</td>
<td>50–300 keV</td>
</tr>
<tr>
<td>GBM/NaI 7</td>
<td>1.5</td>
<td>0.2</td>
<td>50–300 keV</td>
</tr>
<tr>
<td>GBM/NaI 3, 6, and 7</td>
<td>2.1</td>
<td>0.2</td>
<td>50–300 keV</td>
</tr>
<tr>
<td><strong>Swift/BAT</strong></td>
<td><strong>4.0</strong></td>
<td><strong>0.7</strong></td>
<td><strong>50–350 keV</strong></td>
</tr>
<tr>
<td>INTEGRAL-SPI</td>
<td>2.5</td>
<td>0.1</td>
<td>20 keV–10 MeV</td>
</tr>
<tr>
<td><strong>Suzaku-WAM</strong></td>
<td><strong>5.8</strong></td>
<td><strong>0.5</strong></td>
<td><strong>50 keV–5 MeV</strong></td>
</tr>
</tbody>
</table>

*Ackermann et al. 2010*

Compare with Swift 15-150 keV duration Done here at GSFC!
Band function fit to the BGO-0, NaI-6, NaI-7 and BAT data

$\alpha = -0.74$

$\beta = -10$

$E_{pk} = 1907$ keV

$A = 0.039$ /keV/cm$^2$/s

Done here at GSFC!
Spectroscopy of GRB 090510

PL component in addition to phenomenological Band Spectrum

Band-only fit
Band $\alpha = -0.75$
Band $\beta = -2.40$
$E_{pk} = 4.1$ MeV
$A = 0.043$ /keV/cm$^2$/s

$E_{max} = 30.5$ GeV

$E_{max} = 3.43$ GeV

Time-resolved spectra: emergence of a hard power-law component
GRB Jet and Emission Model

Rees, Meszaros, Piran and others ... “standard GRB model”

Synchrotron emission by shocked electrons for prompt and afterglow emission

Internal Shock
- Collisions between different parts of the flow

External Shock
- The flow decelerating into the surrounding medium

GRB
- $\approx 10^{13}$ cm

Afterglow
- $> 10^{16}$ cm
**γγ Opacity and Bulk Lorentz Factor**

Numerical calculation of opacity for $e^+e^-$ pair production opacity

Assumption:
High-energy and target photons from the same internal shocks with radius $R \sim \Gamma^2 c t_v$

Minimum bulk Lorentz factor

$$\tau_{\gamma\gamma}(E = E_{\text{max}}) = 1 \Rightarrow \Gamma = \Gamma_{\text{min}}$$

Band spectrum

$$n(E) = A \left( \frac{E}{100 \text{ keV}} \right)^\alpha \exp \left( -\frac{E(2 + \alpha)}{E_{\text{peak}}} \right); \quad E < E_c$$

$$= A \left( \frac{E_{\text{peak}}(\alpha - \beta)}{100 \text{ keV}(2 + \alpha)} \right)^{\alpha - \beta} \exp(\beta - \alpha) \left( \frac{E}{100 \text{ keV}} \right) ^\beta; \quad E \geq E_c$$

Analytic calculation with a delta-function approximation for the cross-section and for Band spectrum

$$\Gamma_{\text{min}}(E_{\text{max}}) = \left[ \frac{4d_L^2 A}{c^2 t_v} \frac{m_e^2 c^4}{(1 + z)^2 E_{\text{max}} g\sigma_T} \right]^{1 - 2\beta} \left[ \frac{(\alpha - \beta) E_{\text{peak}}}{(2 + \alpha) 100 \text{ keV}} \right]^{\frac{\alpha - \beta}{2 - 2\beta}} \times \exp \left( \frac{\beta - \alpha}{2 - 2\beta} \right) \left( \frac{2m_e c^4}{E_{\text{max}}(1 + z)^2 100 \text{ keV}} \right)^{\frac{\beta}{2 - 2\beta}};$$

for $\Gamma_{\text{min}} > \sqrt{\frac{(1 + z)^2 E_{\text{max}} E_{\text{pk}}(\alpha - \beta)}{2m_e^2 c^4 (2 + \alpha)}}$. 

Gould & Shreder 1966
$\Gamma_{\text{min}}$ for GRB 090510

\[ T_{\gamma}^{+0.6s} - T_{\gamma}^{+0.8s} \]
\[ E_{\text{max}} = 3.43 \text{ GeV} \]

\[ T_{\gamma}^{+0.8s} - T_{\gamma}^{+0.9s} \]
\[ E_{\text{max}} = 30.53 \text{ GeV} \]

\[ \Gamma_{\text{min}} \] Values for the Shortest Timescale Pulses from GRB 090510  

<table>
<thead>
<tr>
<th>$T - T_0$ ,(s)</th>
<th>Spectrum ,</th>
<th>$t_0$ ,(ms)</th>
<th>$E_{\text{max}}$ ,(GeV)</th>
<th>$\Gamma_{\text{min}}^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6–0.8</td>
<td>Band + PL</td>
<td>14 ± 2</td>
<td>3.4</td>
<td>951 ± 38</td>
</tr>
<tr>
<td>0.6–0.8</td>
<td>PL ,b</td>
<td>14 ± 2</td>
<td>3.4</td>
<td>703 ± 34</td>
</tr>
<tr>
<td>0.8–0.9</td>
<td>Band ,c</td>
<td>12 ± 2</td>
<td>30.5</td>
<td>1324 ± 50</td>
</tr>
<tr>
<td>0.8–0.9</td>
<td>Band + PL</td>
<td>12 ± 2</td>
<td>30.5</td>
<td>1218 ± 61</td>
</tr>
<tr>
<td>0.8–0.9</td>
<td>PL ,b</td>
<td>12 ± 2</td>
<td>30.5</td>
<td>1083 ± 88</td>
</tr>
</tbody>
</table>

*Ackermann et al. 2010*
Extended Emission from GRB 090510

Multi-wavelength light curves in $\gamma$ ray, x ray and UV
Smooth power-law evolution of the fluxes are compatible with afterglow model

De Pasquale et al. 2009
GRB Blast Wave Evolution

Coasting blast wave with roughly constant $\Gamma_0$ until total kinetic energy of the “spherical” blast wave = swept-up material (constant density medium $n \text{ cm}^{-3}$)

$$E_k = \frac{4}{3} \pi R^3 n m_p c^2 \Gamma^2$$

With relationship: $R = 2 \Gamma^2 a c t (1 + z)^{−1}$ $a = 1$ coasting

- Deceleration time: $t_{\text{dec}} \approx (1 + z) \left( \frac{3E_k}{32\pi nm_p c^5 \Gamma_0^8} \right)^{1/3} \sim 1.9 (1 + z) (E_{55}/n)^{1/3} \Gamma_3^{-8/3} \text{ s}$

Subsequent evolution in the self-similar phase Blandford-McKee 1976 $a = 4$ after deceleration (adiabatic) $a = 7$ after deceleration (radiative)

Focus on Adiabatic case

- Bulk Lorentz factor: $
\Gamma(t) \approx \left[ \frac{3E_k (1 + z)^3}{32\pi nm_p c^5 a^3 t^3} \right]^{1/8} \sim 763 (1 + z)^{3/8} (E_{55}/n)^{1/8} t_s^{-3/8}$

- Blast wave radius: $R(t) \approx \left[ \frac{3E_k at}{2\pi nm_p c (1 + z)} \right]^{1/4} \sim 1.4 \times 10^{17} (1 + z)^{-1/4} (E_{55}/n)^{1/4} t_s^{1/4} \text{ cm}$
Forward Shock in the GRB Blast Wave

Forward shock evolves as $\Gamma(t)$

- Energy injection rate in the forward shock: $e_{\text{shock}} = 4 \pi n m_p c^2 \Gamma^2$

A fraction $\varepsilon_B = u_B / e_{\text{shock}}$ is converted to magnetic field

- Magnetic field in the FS: $B'(t) \approx \Gamma(t) \sqrt{32 \pi \varepsilon_B n m_p c^2} \sim 300 (1 + z)^{3/8} \varepsilon_B^{1/2} (E_{55} n^3)^{1/8} t_s^{-3/8}$ G

A fraction $\varepsilon_e$ of $e_{\text{shock}}$ is injected in shocked electrons forming a non-thermal spectrum above

$$\gamma'_{m,e}(t) = \eta_e (m_p / m_e) \Gamma(t)$$

$$\varepsilon_e = \xi_e \eta_e \frac{k-1}{k-2} \frac{1-(\gamma'_{m,e} / \gamma'_{\text{sat},e})^{k-2}}{1-(\gamma' / \gamma'_{\text{sat},e})^{k-1}} ; k \neq 2$$

$n_e = n_p$ in the pre-shocked fluid

$\xi_e$ is the fraction accelerated
Synchrotron Afterglow Model - I

Synchrotron radiation by shock-accelerated electrons in the FS

• Synchrotron cooling time for electrons: \( t'_{\text{syn}}(\gamma'_e) = \frac{6\pi m_e c}{\sigma_T B^2 \gamma'_e} \)

Equate cooling time to dynamic time \( t'_{\text{dyn}} = t\Gamma/(1+z) \) and solve for \( \gamma'_e \)

• Synchrotron cooling break: \( \gamma'_{c,e}(t) \sim 12(1+z)^{-1/8} \epsilon_B^{-1} (E_{55}^3 n^5)^{-1/8} t_s^{1/8} \)

Fast cooling - all electrons cool within dynamic time: \( \gamma'_c, e < \gamma'_m, e \)
Slow cooling - high-energy electrons cool within dynamic time: \( \gamma'_c, e > \gamma'_m, e \)

• Cooling causes a break in the electron spectrum: \( n(\gamma'_e) \propto \begin{cases} \gamma'^{-k}; & \gamma'_e < \gamma'_c, e \\ \gamma'^{-k-1}; & \gamma'_e > \gamma'_c, e \end{cases} \)

(and a break in synchrotron spectrum)

• Synchrotron frequency:

\[
h\nu_{m,e} \approx 7.7(1+z)^{1/2} \gamma_e^2 (\epsilon_B E_{55})^{1/2} t_s^{-3/2} \text{ GeV},
\]
\[
h\nu_{\text{sat},e} \approx 180(1+z)^{-5/8} \phi_e^{-1} (E_{55}/n)^{1/8} t_s^{-3/8} \text{ GeV},
\]
\[
h\nu_{c,e} \approx 0.5(1+z)^{-1/2} \epsilon_B^{-3/2} (E_{55} n^2)^{-1/2} t_s^{-1/2} \text{ eV}.
\]
Synchrotron Afterglow Model - II

Synchrotron flux normalization

- Total number of emitting electrons in the blast wave: \( N_e(t) = \frac{4}{3} \pi R^3(t) \xi_e n \)
- Synchrotron power from each electron: \( P_e(\gamma'_e) = \frac{c\Omega_T}{6\pi} B^2 \gamma'_e^2 \)
- Total synchrotron flux at maximum from the blast wave at \( \nu_{m,e} \)

\[
F_{\nu,e}^{\text{max}} = \frac{N_e(t) P_e(\gamma'_{m,e})}{4\pi d_L^2} \frac{\Gamma^2(t)}{h\nu_{m,e}} \frac{\xi_e E_{55}(\epsilon_B n)^{1/2}}{1+z} \sim 52 \frac{\xi_e E_{55}(\epsilon_B n)^{1/2}}{(1+z) d_{28}^2} \text{Jy}
\]

Fast-cooling spectrum: \( \nu_{c,e} < \nu_{m,e} \); Slow-cooling spectrum: \( \nu_{c,e} > \nu_{m,e} \)

- Transition time from fast-to-slow \( \nu_{c,e} = \nu_{m,e} \): \( t_{0,e} \sim 1.5 \times 10^{10} (1+z)(\epsilon_B \eta_e)^2 n E_{55} \text{ s} \)
- Synchrotron break frequencies and flux evolve with time
- Different parts of the spectra evolve differently \( \Rightarrow \) closure relations
GRB Afterglow - Synchrotron Spectra

Fast cooling: $\nu_m > \nu_c$

$F_\nu \propto \nu^{-\beta} t^{-\alpha}$ closure relations

$\nu_c < \nu < \nu_m : F_\nu \propto \nu^{-1/2} t^{-1/4}$

$\nu > \nu_m > \nu_c : F_\nu \propto \nu^{-p/2} t^{-3/4(p-2/3)}$

$p$-particle spectral index: $\frac{dN}{dE} \propto E^{-p}$

Slow cooling: $\nu_c > \nu_m$

$F_\nu \propto \nu^{-\beta} t^{-\alpha}$ closure relations

$\nu_m < \nu < \nu_c : F_\nu \propto \nu^{-(p-1)/2} t^{-3/4(p-1)}$

$\nu > \nu_c > \nu_m : F_\nu \propto \nu^{-p/2} t^{-3/4(p-2/3)}$
Jet break and Spectral Changes

Spherical blast wave approximation is valid as long as $\theta_{jet} > \theta_{view} \sim 1/\Gamma(t)$

Edges of the jet becomes visible at a time $t = t_{jet}$ when $\Gamma(t_{jet}) \sim 1/\theta_{jet}$

Sari, Piran & Halpern 1999

• Jet-break time: $t_{jet} \approx 10^5 (1 + z) (E_{55}/n)^{1/3} \theta_{-1}^{8/3}$ s

$\nu_m \propto t^{-2} [t^{-3/2}]$

$\nu_c \propto \text{const.} [t^{-1/2}]$

$F_v^{\text{max}} \propto t^{-1} [\text{const.}]$

• Modified closure relations after break

$\nu_m < \nu < \nu_c: F_v \propto \nu^{-(p-1)/2} t^{-p} \left[ t^{-3/4(p-1)} \right]$ $\nu > \nu_c > \nu_m: F_v \propto \nu^{-p/2} t^{-p} \left[ t^{-3/4(p-2/3)} \right]$ Acromatic break in Flux decay

index $\Rightarrow \alpha \text{ and } \beta \text{ are not related}$

Collimation-corrected absolute jet energy: $E_{jet} \approx \frac{1}{2} \theta_{jet}^2 E_k; \theta_{jet} \ll 1$
Leptonic-Hadronic Synchrotronon Model

Both electrons and ions are accelerated in the Forward shock

![Ion spectrum](image)

![Electron spectrum](image)

Scaling relations for proton- to ion-synchrotron radiation

\[
\begin{align*}
\nu_{m,A} &= Z(\eta_A/\eta_e)^2(m_e/m_p)^3 \nu_{m,e}, \\
\nu_{c,A} &= (A^6/Z^7)(m_p/m_e)^5 \nu_{c,e}, \\
F_{\nu,A}^{\text{max}} &\sim \frac{k_1 - 1}{k_2 - 1 + \eta_A^{k_1 - 1}} \frac{\xi_A}{\xi_e} \frac{Z^3 m_e}{A^2 m_p} F_{\nu,e}^{\text{max}} \\
\end{align*}
\]

- Crucial parameters: \( \epsilon_B, \eta_A, \eta_e, k \) and \( k_2 \) are fitted from data
- Fraction of jet energy: \( \epsilon_A \) and \( \epsilon_e \) are calculated from required spectra
Modeling GRB 090510 Data

Use closure relations $F_{\nu} \propto \nu^{-\beta} t^{-\alpha}$ to determine $\beta$ and $k$ or $k_2$

Note: $e$-synchrotron model alone cannot satisfy the closure relations

- XRT light curve: $t^{-0.74 \pm 0.03}$ in between $\sim 100$ s and 1.4 ks
  - Model with $e$-synchrotron in the fast-cooling and for $\nu_{XRT} > \nu_{m,e} > \nu_{c,e}$
  - $k = (4/3)\alpha_{XRT} + 2/3 = 1.65 \pm 0.04$ ; $\beta_{XRT} = k/2 = 0.83 \pm 0.02$

- LAT light curve: $t^{-1.38 \pm 0.07}$ in between $\sim 0.3$ s and 100 s
  - Model with $p$-synchrotron in the slow-cooling and for $\nu_{m,p} < \nu_{LAT} < \nu_{c,p}$
  - $k_2 = (4/3)\alpha_{\gamma} + 1 = 2.84 \pm 0.09$ ; $\beta_{\gamma} = (k_2 - 1)/2 = 0.92 \pm 0.05$

- $\beta_{\gamma}$ needs to be compatible with measured LAT photon index (and it is)

- Parameters such as $n_{ISM}$ and $\Gamma_0$ are mainly constrained by $t_{\text{dec}} \leq 0.3$ s

- Parameters such as $E_{k,iso}$, $\varepsilon_B$, $\eta_e$, $\eta_p$ are set to produce required fluxes

- Parameters $\varepsilon_e$, $\varepsilon_p$ are calculated from other parameters and constrained $< 1$

- UVOT light curve is constrained by XRT ($e$-synchrotron)

- BAT light curve can not be fitted $\Rightarrow$ continued central engine activity
Leptonic-Hadronic Synchrotron Spectra

Protons and electrons

- $p$ is always slow-cooling
- $e$ shifts from fast- to slow- cooling in $2 \times 10^6$ s

$p$ - synchrotron
$k_2 = 2.84$

$$\nu F_\nu \propto \begin{cases} \nu^{4/3} ; \nu < \nu_{m,p} \\ \nu^{0.08} ; \nu \geq \nu_{m,p} \end{cases}$$

$e$ - synchrotron
$k = 1.65$

$$\nu F_\nu \propto \begin{cases} \nu^{4/3} ; \nu < \nu_{c,e} \\ \nu^{1/2} ; \nu_{m,e} > \nu \geq \nu_{c,e} \\ \nu^{0.18} ; \nu \geq \nu_{m,e} \end{cases}$$

LAT emission is dominated by $p$-synchrotron with photon spectrum
$$\propto \nu^{-1.92}$$
Compatible with data
Light Curves from Afterglow Modeling

Multiwavelength light curves from combined leptonic-hadronic modelling

Solid lines: $p$-synchrotron, Dashed lines: $e$-synchrotron

$E_k = 2 \times 10^{55}$ erg
$n = 3 \text{ cm}^{-3}$
$\Gamma_0 = 2400$
$\epsilon_B = 0.3$
$\epsilon_p = 0.5$
$\epsilon_e = 10^{-4}$
$\eta_e = 20(m_e/m_p)$
$\eta_p = 5000$
$k = 1.65 \pm 0.04$
$k_2 = 2.84 \pm 0.09$

Razzaque 2010
Absolute GRB Jet Energy

Ratio of gamma-ray to kinetic energy

\[ E_{\gamma, \text{iso}} \sim 10^{53} \text{ erg} \]
\[ E_{\gamma, \text{iso}} / E_{k, \text{iso}} \sim 0.01 \]

Collimation-corrected energy follows from jet-break time

\[ t_{\text{jet}} \approx 10^5 (1 + z) (E_{55}/n)^{1/3} \theta_{-1}^{8/3} \text{ s} \]

Sari, Piran & Halpern 1999

Jet-break time during the Earth Occultation: 1.4 ks < \( t_{\text{jet}} \) < 5.1 ks

\( \Rightarrow \) Jet opening angle: 1 degree < \( \theta_{\text{jet}} \) < 1.5 degree

\( \Rightarrow \) Absolute jet energy: \((3-7) \times 10^{51}\) erg

Is there an absolute maximum? \( 10^{53} \) erg \( \Rightarrow \) \( \theta_{\text{jet}} \sim 6 \) degree
Build your own Afterglow Model and fit Multiwavelength Data